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# Calculation of the induced electromagnetic field created by an arbitrary current distribution located outside a conductive cylinder

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**Abstract.** The present work approaches the problem of determining, by analytical solutions, the electromagnetic field created by an arbitrary distribution of alternating currents placed outside an infinitely long non-magnetic conductive cylinder. Making use of the dyadic Green function method, the electromagnetic field outside the cylinder is expressed as the sum of the field in free space and the field created by the currents induced in the conductive cylinder. The general results we obtained are particularized for the analytical solutions describing the operation of an eddy current transducer with rotating magnetic field for the case of a conductive cylinder non-coaxial with the current source.

## 1. Introduction

In eddy current non-destructive control theory the work of Dodd and Deeds [1] has represented the first complete analytical treatment for two situations frequently occurring in practice: a circular coil placed on a conductive half-space, and an infinite conductive cylinder coaxial with the coil. The proposed method can be applied to axial symmetries leading to closed-form solutions by means of the Fourier–Bessel integrals.

To model asymmetrical transducer operation, Bessner and Sablick [2] have determined the electric field inside a half-space conductor for an external current source of an arbitrary shape. Subsequent developments have permitted the modelling of the response of eddy current transducers with various shapes, even in the presence of material discontinuities [3, 4].

A possible method to solve this problem consists in the utilization of the dyadic Green functions [5, 6]. This procedure has the advantage of enabling the solution to be written in a formal form for any current source shape, simultaneously reducing the number of zones into which the domain of interest has to be divided, and the number of constants to be determined from the boundary conditions.

This work deals with the determination, by analytical solutions, of the electromagnetic field created by an arbitrary alternating current distribution external to a conductive cylinder. Using the dyadic Green function method, the electromagnetic field has been calculated for every position of the cylinder relative to the current source. The obtained general results are particularized for the analytical solutions describing the operation of an eddy current transducer with rotating magnetic field, obtained in [7] for the case of a coaxial cylinder. The generality of the method enables also the analysis of the influence of eccentricity of the conductive cylinder, located inside the transducer, on the transducer response.

The eddy current transducer with rotating magnetic field permits us to detect long material discontinuities placed almost parallel to the cylinder's axis. In this type of transducer, the magnetic field rotating around the transducer's axis has its rotating frequency equal to the excitation current frequency and is scanning the lateral surface of the inspected product.

## 2. Theory

Let us consider an infinitely long conductive cylinder of radius $r_0$, placed inside a current source of an arbitrary shape (see figure 1). The cylinder has electric conductivity $\sigma$ and magnetic permeability $\mu = \mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$, being homogeneous, isotropic and with linear properties. The cylinder represents the $\Omega_1$ domain, delimited by the surface $r = r_0$. The surroundings represent the $\Omega_2$ domain, with zero electric conductivity and $\mu = \mu_0$. Within the $\Omega_2$ domain a current source is located, with the distribution $\boldsymbol{J}(\boldsymbol{x})$ sinusoidal in time and of an arbitrary form. For frequency lower than 10 MHz, the domain of interest in the eddy current control, the displacement current can be neglected [8].

  

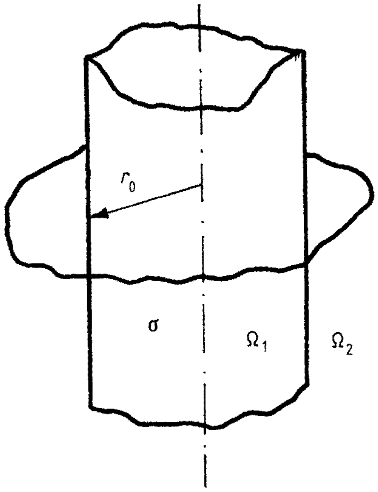


**Figure 1.** Conducting cylinder inside an arbitrary current source.

In the absence of the conductive cylinder, the electric field $\boldsymbol{E}_0$ created by the source satisfies the equation [8]:

$$\nabla^2 \boldsymbol{E}_0 = -\mathrm{j}\omega\mu_0 \boldsymbol{J} \tag{1}$$

where $\mathrm{j} = \sqrt{-1}$, $\omega$ is the angular frequency of the alternating current in the source.

Given the arbitrary shape of the current source, $\boldsymbol{E}_0$ can be determined by using the dyadic Green function $\mathbf{G}(\boldsymbol{x}, \boldsymbol{x}')$, solution of the equation [9]:

$$\nabla^2 \mathbf{G}_0(\boldsymbol{x}, \boldsymbol{x}') = -\delta(\boldsymbol{x}, \boldsymbol{x}')\mathbf{I} \tag{2}$$

where $\boldsymbol{x}$ represents the position vector of a point belonging to the source, $\boldsymbol{x}'$ is the position vector of an arbitrary point in the space, $\delta(\boldsymbol{x}, \boldsymbol{x}')$ is the Dirac functional and $\mathbf{I}$ is the identity dyad.

The electric field $\boldsymbol{E}_0$ is defined as [10]:

$$\boldsymbol{E}_0(\boldsymbol{x}) = \mathrm{j}\omega\mu_0 \iiint_{V source} \mathbf{G}_0(\boldsymbol{x}, \boldsymbol{x}') \cdot \boldsymbol{J}(\boldsymbol{x}')\,\mathrm{d}\boldsymbol{x}' \tag{3}$$

the integration extending over the source volume.

The symmetry of the $\Omega_1$ domain dictates the choice of a cylindrical coordinate system with the unit vectors $\boldsymbol{u}_r$, $\boldsymbol{u}_\theta$, $\boldsymbol{u}_z$ having the O$z$ axis parallel to the cylinder axis. $\mathbf{G}(\boldsymbol{x}, \boldsymbol{x}')$ must have finite values along the O$z$ axis and vanish at $r \to \infty$. According to [9], the Green function satisfying these conditions has the form:

$$\mathbf{G}_0(\boldsymbol{x}, \boldsymbol{x}') = \frac{\mathbf{I}}{|\boldsymbol{x} - \boldsymbol{x}'|} \tag{4}$$

where, for the chosen coodinate system, $\mathbf{I} = \boldsymbol{u}_r\boldsymbol{u}_r + \boldsymbol{u}_\theta\boldsymbol{u}_\theta + \boldsymbol{u}_z\boldsymbol{u}_z$. According to [8]:

$$\frac{1}{|\boldsymbol{x} - \boldsymbol{x}'|} = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} I_m(|h|r_<)K_m(|h|r_>) \times \exp[\mathrm{j}h(z - z') + \mathrm{j}m(\theta - \theta')]\,\mathrm{d}h \tag{5}$$

where we used the notations: $r_< = \min(r, r')$; $r_> = \max(r, r')$; $I_m$ is the modified Bessel function of rank I and $m$th order; $K_m$ is the Bessel function of rank II and $m$th order.

Introducing (4) and (5) into (3) gives the components of the electric field $\boldsymbol{E}_0$ in the zone delimitated by the current source of an arbitrary shape and the lateral surface of the conducting cylinder (figure 1):

$$E_{0r}(r, \theta, z) = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}h \left[ -\mathrm{j}C_1 \frac{h}{|h|} I'_m(|h|r) - \frac{\mathrm{j}m}{h^2} C_2 \frac{I_m(|h|r)}{r} \right] \mathrm{e}^{\mathrm{i}(m\theta + hz)} \tag{6}$$

$$E_{0\theta}(r, \theta, z) = \sum_{m=-\infty}^{-\infty} \int_{-\infty}^{+\infty} \mathrm{d}h \left[ C_1 \frac{m}{h} \frac{I_m(|h|r)}{r} + \frac{|h|}{h^2} C_2 I'_m(|h|r) \right] \mathrm{e}^{\mathrm{i}(m\theta + hz)} \tag{7}$$

$$E_{0z}(r, \theta, z) = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}h\,[C_1 I_m(|h|r)]\,\mathrm{e}^{\mathrm{j}(m\theta + hz)} \tag{8}$$

where:

$$C_1(m, h) = \frac{\mathrm{j}\omega\mu_0}{4\pi^2} \iiint_{V source} K_m(|h|r)\,\mathrm{e}^{-\mathrm{j}(m\theta + hz)} \times \boldsymbol{u}_z \cdot \boldsymbol{J}(r, \theta, z)\,\mathrm{d}V \tag{9}$$

$$C_2(m, h) = \frac{\mathrm{j}\omega\mu_0}{4\pi^2} \iiint_{V source} [\nabla \times \boldsymbol{u}_z K_m(|h|r)\,\mathrm{e}^{-\mathrm{i}(m\theta + hz)}] \cdot \boldsymbol{J}(r, \theta, z)\,\mathrm{d}V \tag{10}$$

and the prime represents the derivative of $I_m(|h|r)$ with respect to the variable.

The presence of conductive cylinder inside the current source will result in the appearance of a supplementary electric field produced by the induced eddy currents, which in the region $\Omega_2$ is the solution of the equation:

$$\nabla^2 \boldsymbol{E}^1 = 0. \tag{11}$$

The field $\boldsymbol{E}^1$ has to vanish for $r \to \infty$. In the chosen coordinate system, the components of $\boldsymbol{E}^1$ are:

$$E_r^1(r, \theta, z) = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}h \left[ -\mathrm{j}F_1 \frac{h}{|h|} K'_m(|h|r) - \frac{\mathrm{j}m}{h^2} F_2 \frac{K_m(|h|r)}{r} \right] \mathrm{e}^{\mathrm{j}(m\theta + hz)} \tag{12}$$

$$E_\theta^1(r, \theta, z) = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}h \left[ F_1 \frac{m}{h} \frac{K_m(|h|r)}{r} + \frac{|h|}{h^2} F_2 K'_m(|h|r) \right] \mathrm{e}^{\mathrm{i}(m\theta + hz)} \tag{13}$$

$$E_z^1(r, \theta, z) = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}h\,[F_1 K_m(|h|r)]\,\mathrm{e}^{\mathrm{i}(m\theta + hz)} \tag{14}$$

where the functions $F_1(m, h)$ and $F_2(m, h)$ have to be determined from the conditions of continuity for the electromagnetic field on the boundary of $\Omega_1$.

The electric field in the $\Omega_1$ domain is $\boldsymbol{E}_1$, satisfying the equation:

$$\nabla^2 \boldsymbol{E}_1 + \mathrm{j}\omega\mu_0\sigma \boldsymbol{E}_1 = 0 \tag{15}$$

having to be finite on the axis of the system.

The components of $\boldsymbol{E}_1$ are:

$$E_{1r}(r,\theta,z) = \sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}h\left[-\mathrm{j}A_1\frac{h}{a}I'_m(ar) - \frac{\mathrm{j}m}{a^2}A_2\frac{I_m(ar)}{r}\right]\mathrm{e}^{\mathrm{j}(m\theta+hz)} \tag{16}$$

$$E_{1\theta}(r,\theta,z) = \sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}h\left[A_1\frac{mh}{a^2}\frac{I_m(ar)}{r} + \frac{A_2}{a}I'_m(ar)\right]\mathrm{e}^{\mathrm{j}(m\theta+hz)} \tag{17}$$

$$E_{1z}(r,\theta,z) = \sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}h\ [A_1I_m(ar)]\ \mathrm{e}^{\mathrm{j}(m\theta+hz)} \tag{18}$$

where $a = (h^2 - \mathrm{j}\omega_0\sigma)^{1/2}$, $A_1(m,h)$ and $A_2(m,h)$ being functions which are to be determined from the conditions of continuity for the electromagnetic field on the boundary of $\Omega_1$.

The electric field in the $\Omega_2$ domain is $\boldsymbol{E}_2$, the sum of the electric field in the absence of the cylinder, and the field due to the eddy currents:

$$\boldsymbol{E}_2 = \boldsymbol{E}_0 + \boldsymbol{E}^1. \tag{19}$$

Within the space delimited by the innermost part of the source and the surface of the cylinder, the components of $\boldsymbol{E}_2$ will be:

$$E_{2r}(r,\theta,z) = -\mathrm{j}\sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left[\frac{h}{|h|}(C_1(m,h)I'_m(|h|r) + F_1(m,h)K'_m(m,h)) + \frac{m}{h^2r}(C_2(m,h)I_m(|h|r) + F_2(m,h)K_m(|h|r))\right]\mathrm{e}^{\mathrm{j}(m\theta+hz)}\right]\mathrm{d}h \tag{20}$$

$$E_{2\theta}(r,\theta,z) = \sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\left[\frac{m}{hr}(C_1(m,h)I_m(|h|r)) + F_1(m,h)K_m(m,h) + \frac{|h|}{h^2}(C_2(m,h)I'_m(|h|r) + F_2(m,h)K'_m(|h|r))\right]\mathrm{e}^{\mathrm{j}(m\theta+hz)}\right]\mathrm{d}h \tag{21}$$

$$E_{2z}(r,\theta,z) = \sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}[(C_1(m,h)I_m(|h|r) + F_1(m,h)K_m(m,h))\ \mathrm{e}^{\mathrm{j}(m\theta+hz)}]\,\mathrm{d}h. \tag{22}$$

The functions $A_1(m,h)$, $A_2(m,h)$, $F_1(m,h)$, $F_2(m,h)$ are determined from the conditions of continuity on the surface separating the domains $\Omega_1$ and $\Omega_2$:

$$\begin{aligned}\boldsymbol{E}_1\times\boldsymbol{u}_r|_{r=r_0} &= \boldsymbol{E}_2\times\boldsymbol{u}_r|_{r=r_0}\\ \boldsymbol{H}_1\times\boldsymbol{u}_r|_{r=r_0} &= \boldsymbol{H}_2\times\boldsymbol{u}_r|_{r=r_0}\\ \boldsymbol{B}_1\cdot\boldsymbol{u}_r|_{r=r_0} &= \boldsymbol{B}_2\cdot\boldsymbol{u}_r|_{r=r_0}\end{aligned} \tag{23}$$

where

$$\begin{aligned}\boldsymbol{B}_1(r,\theta,z) &= \frac{1}{\mathrm{j}\omega}\nabla\times\boldsymbol{E}_1(r,\theta,z)\\ \boldsymbol{B}_2(r,\theta,z) &= \frac{1}{\mathrm{j}\omega}\nabla\times\boldsymbol{E}_2(r,\theta,z)\end{aligned} \tag{24}$$

and $\boldsymbol{B} = \mu\boldsymbol{H}$.

By replacing the relations (16), (17), (18), (20), (21), (22) and (24) in the system (23), one obtains finally:

$$A_1 = \frac{C_2m}{|h|r_0^2ahI'_m(|h|r_0)[K'_m(|h|r_0) - fK_m(|h|r_0)]} \tag{25}$$

$$A_2 = -\frac{C_2}{|h|r_0I_m(ar_0)[K'_m(|h|r_0) - fK_m(|h|r_0)]} \tag{26}$$

$$F_1 = -C_1\frac{I_m(|h|r_0)}{K_m(|h|r_0)} + C_2\frac{m}{|h|r_0^2ah}\frac{1}{K_m(|h|r_0)} \times\frac{I_m(|h|r_0)}{I'_m(|h|r_0)[K'_m(|h|r_0) - fK_m(|h|r_0)]} \tag{27}$$

$$F_2 = C_2\frac{-I'_m(|h|r_0) + fI_m(|h|r_0)}{K'_m(|h|r_0) - fK_m(|h|r_0)} \tag{28}$$

where we denoted:

$$f = \frac{m^2}{r_0^2a|h|}\left(1 - \frac{h^2}{a^2}\right)\frac{I_m(ar_0)}{I'_m(ar_0)} + \frac{|h|}{a}\frac{I'_m(ar_0)}{I_m(ar_0)}. \tag{29}$$

By replacing the relations (25) and (26) with the notation (29) in relations (16), (17) and (18), one obtains the closed forms of the electric field components inside the domain $\Omega_2$.

Using the relations (24) one can determine the magnetic flux density in the two domains.

The obtained results can be particularized for the case of the infinitely long conductive cylinder placed concentrically with a circular turn supplied by an alternative current, the situation described in [1].

Supposing the circular turn of radius $R$ is placed in the plane $z = z_0$; the distribution of the alternating current source can be written as:

$$J(r,\theta,z) = I_0\delta(r-R)\delta(z-z_0)\boldsymbol{u}_\theta \tag{30}$$

where $I_0$ is the current amplitude in the source.

By replacing (30) in the expressions giving the components of the electric field inside the two domains, one obtains results identical to those obtained in [1] for the case of the perfectly homogeneous cylinder.

The method proposed here needs no conditions of continuity on the boundary delimitating the current source, which reduces the size of the calculation and enables us to deal with geometries of more complicated shape of the current source.

## 3. Application: conductive cylinder in rotating magnetic field

Cylinder conductive products, like wires, bars and pipes, are often non-destructively examined by eddy current methods in order to detect superficial and sub-superficial flaws. As a result of the manufacturing technology, it is possible that the obtained product presents long material discontinuities placed almost parallel to its axis.

These discontinuities can appear on drawn wires if the die is wrong, or in pipes welded after their generatrix if the welding is discontinuous; or in rolled products if the rolls are worn out. For this type of product it is important to detect both short and long discontinuities. There are now, two procedures enabling this type of control:

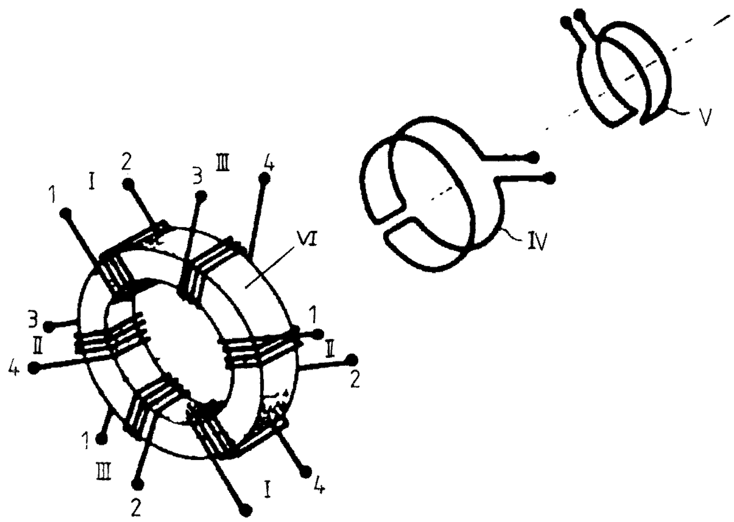


**Figure 2.** Basic diagram of the transducer with rotating magnetic field: I, II, III—excitation windings; IV, V—pick-up coils; VI—high-permeability ferrite core.

—control with an absolute encircling transducer, able to detect also long discontinuities extending for the whole product length, yet it has the shortcomings of poor sensitivity, and high noise resulting, mainly, from the vibration of the controlled product inside the transducer, and

—control with a laid-on transducer rotating around the inspected product, with the advantages of good sensitivity and reduced noise, yet with the disadvantages of a complicated construction, a limited control velocity and the difficulty of controlling high-temperature products.

An alternative approach which we have proposed [11] is to use a stationary transducer with rotating magnetic field coupled with a pick-up coil encircling the product. This has the advantage of a simple construction, due to the fact that the external surface of the product under inspection is scanned by stationary coils.

The basic diagram of the transducer is presented in figure 2.

On a high-permeability ferrite torus three pairs of coils are wound, placed at $2\pi/3$ apart. The windings of a pair are connected in series–opposition and are supplied by three alternating currents of the same frequency and amplitude, dephased by $2\pi/3$. Given the connection of the windings, three leakage alternating magnetic fields of the same amplitude and dephased by $2\pi/3$ will be produced. Their resultant is a rotating magnetic field. This construction is basically equivalent with that of three identical coils placed $2\pi/3$ apart and supplied by three-phase currents (figure 3).

This section is dedicated to the determination of the expressions of the electromotive force induced in the pick-up coil of the eddy current transducer with rotating magnetic field, as a particular case of the above theory.

According to [12], a system like that in figure 3 is equivalent to an infinite sequence of longitudinal and transversal current sheets of the same amplitude (figure 4).

In the cylindrical coordinate system $\rho$, $\varphi$, $z$ with the O$z$ axis parallel to the current sheets axis, and the unit vectors

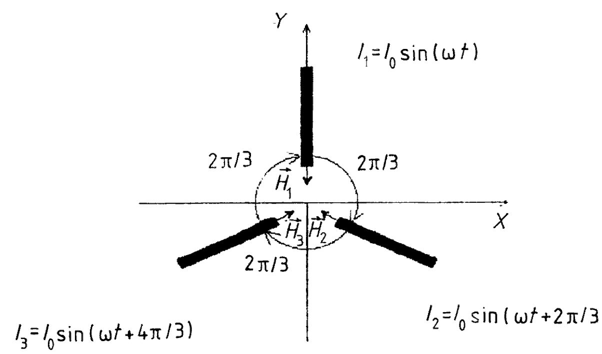


**Figure 3.** Principle of rotating magnetic field generation.

$\boldsymbol{u}_\rho$, $\boldsymbol{u}_\varphi$, $\boldsymbol{u}_z$, the current source can be written as:

$$\boldsymbol{J}(\rho, \varphi, z) = i_z \boldsymbol{u}_z + i_\varphi \boldsymbol{u}_\varphi \qquad (31)$$

where, according to [12]:

$$i_z = \frac{4}{\pi} \hat{i}_z \sum_{s=1}^{\infty} \frac{1}{s} \sin(\alpha L) \cos(\alpha z) \sin(p\varphi) \delta(\rho - R) \qquad (32)$$

$$i_\varphi = -\frac{2R}{H} \hat{i}_z \sum_{s=1}^{\infty} \frac{1}{s} \sin(\alpha L) \sin(\alpha z) \frac{\cos(p\varphi)}{p} \delta(\rho - R) \qquad (33)$$

where $\hat{i}_z$ represents the amplitude of the alternative current with the angular frequency $\omega$ inside the sheets, $R$ is the current sheet radius, $2L$ is the length of the longitudinal current sheet; $2H$ is the distance along the O$z$ axis at which the $z$ components of the magnetic induction created by the transducer vanishes, i.e. the zone within which the transducer is sensitive to the material discontinuities; $\alpha = s\pi/2H$, with $s = 1, 3, 5, \ldots$; $p$ the number of transducer pole pairs. For our transducer, $p = 3$.

An increased number of pole pairs will result in the improvement of the angular distribution uniformity of the

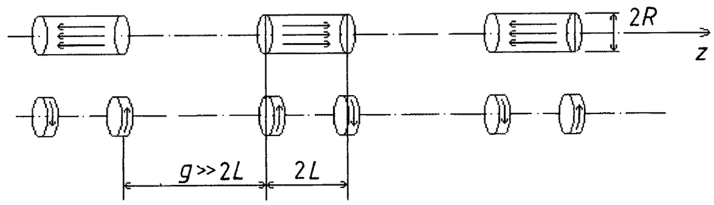


**Figure 4.** Substitution of the emission part of the eddy current transducer by an infinite sequence of axial and longitudinal current sheets.

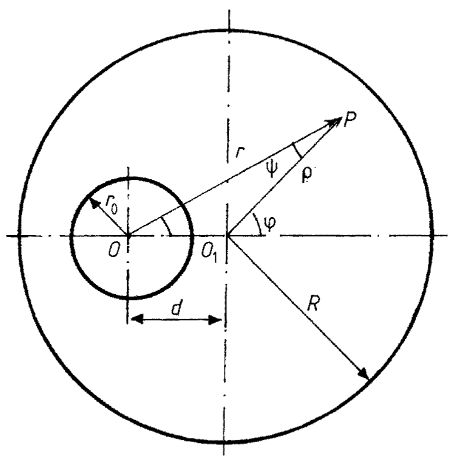


**Figure 5.** Conducting cylinder placed eccentrically inside the current sheets.

rotating magnetic field amplitude inside the transducer, yet its physical realization is more complicated.

The components of the electric field in the conductive cylinder and within the region between the cylinder and the current sheets, where the transducer coils are placed, for the case of a cylinder coaxial with the current sheets, have the expressions obtained by replacing (31) in (16), (17), (18) and (20), (21), (22) respectively.

The functions $C_1(m,h)$ and $C_2(m,h)$ appearing in the expressions of $A_1(m,h)$; $A_2(m,h)$; $F_1(m,h)$ and $F_2(m,h)$ are given by:

$$C_1(m,h) = \omega\mu_0 R K_m(|h|R)\hat{i}_z \frac{4}{\pi}\frac{\sin(\alpha L)}{s} \times[\delta(p-m)-\delta(p+m)][\delta(\alpha+h)+\delta(\alpha-h)] \quad (34)$$

$$C_2(m,h) = \omega\mu_0 R K'_m(|h|R)\hat{i}_z \frac{2R}{H}\frac{\sin(\alpha L)}{p} \times[\delta(p-m)+\delta(p-m)][\delta(\alpha-h)-\delta(\alpha+h)]. \quad (35)$$

In the case when the cylinder axis is not identical to the current sheet axis, two cylindrical coordinate systems are used, namely the system $\rho, \varphi, z$ attached to the current sheets, i.e. to the transducer, and the system $r, \theta, z$ attached to the conductive cylinder (figure 5).

In order to write the conditions of continuity on the lateral surface of the conductive cylinder, we need the components of the electric fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the coordinate system attached to the cylinder.

For a point P placed within the zone between the cylinder and the current sheets, one can write [9]:

$$\mathrm{e}^{\mathrm{j}q\psi} I_q(|h|\rho) = \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{j}m\theta} I_m(|h|x_<) I_{m+q}(|h|x_>) \quad (36)$$

where $\rho, \psi, \theta$ have the meaning given in figure 5, $d$ is the distance between the axes, $x_< = \min(r,d)$ and $x_> = \max(r,d)$.

Using the relation (36), the components of the electric field inside the conductor can be written as:

$$E_{1r} = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\left[A_1\frac{\alpha}{a}I'_m(ar) + A_2\frac{\alpha}{a}\frac{I_m(ar)}{r}\right] \times \sin(m\theta)\sin(\alpha z) \quad (37)$$

$$E_{1\theta} = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\left[A_1\frac{\alpha m}{a^2}\frac{I_m(ar)}{r} + A_2\frac{1}{a}I'_m(ar)\right] \times \cos(m\theta)\sin(\alpha z) \quad (38)$$

$$E_{1z} = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} A_1 I_m(ar)\sin(m\theta)\cos(\alpha z). \quad (39)$$

The components of the electric field inside the region between the cylinder and the current sheets are:

$$E_{2r}(r,\theta,z) = \sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\left[C_1 I'_m(\alpha r) + F_1 K'_m(\alpha r) + \frac{m}{\alpha^2 r}(C_2 I_m(\alpha r) + F_2 K_m(\alpha r))\right]\sin(m\theta)\sin(\alpha z) \quad (40)$$

$$E_{2\theta}(r,\theta,z) = \sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\frac{1}{\alpha}\left[\frac{m}{r}[C_1 I_m(\alpha r) + F_1 K_m(\alpha r)] + \frac{m}{\alpha^2 r}(C_2 I'_m(\alpha r) + F_2 K'_m(\alpha r))\right]\cos(m\theta)\sin(\alpha z) \quad (41)$$

$$E_{2\theta}(r,\theta,z) = \sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}[C_1 I_m(\alpha r) + F_1 K_m(\alpha r)] \times \sin(m\theta)\cos(\alpha z) \quad (42)$$

where:

$$A_1 = \frac{2\hat{i}_z\omega\mu_0 R^2 K'_p(\alpha R)}{\alpha^2 r_0^2 a H}\sin(\alpha L)\frac{m\varepsilon_m}{p} \times \frac{(I_{m-p}(\alpha d) - I_{m+p}(\alpha d))}{I'_m(\alpha r_0)} \times\left(K'_m(\alpha r_0) - \left(\frac{m^2}{r_0^2 a\alpha}\left(1-\frac{\alpha^2}{a^2}\right)\frac{I_m(ar_0)}{I'_m(ar_0)} + \frac{\alpha I'_m(ar_0)}{aI_m(ar_0)}\right)K_m(ar_0)\right)^{-1} \quad (43)$$

$$A_2 = -\frac{2\hat{i}_z\omega\mu_0 R^2 K_p'(\alpha R)}{\alpha r_0 H p}\sin(\alpha L)\varepsilon_m \times\frac{(I_{m-p}(\alpha d) - I_{m+p}(\alpha d))}{I_m'(\alpha r_0)} \times\left(K_m'(\alpha r_0) - \left(\frac{m^2}{r_0^2 a\alpha}\left(1-\frac{\alpha^2}{a^2}\right)\frac{I_m(ar_0)}{I_m'(ar_0)} + \frac{\alpha I_m'(ar_0)}{aI_m(ar_0)}\right)K_m(ar_0)\right)^{-1} \tag{44}$$

$$C_1 = \frac{4\hat{i}_z\omega\mu_0 R K_p(\alpha R)}{\pi s} \times\sin(\alpha L)\varepsilon_m(I_{m-p}(\alpha d) + I_{m+p}(\alpha d)) \tag{45}$$

$$C_2 = \frac{2\hat{i}_z\omega\mu_0 R^2 K_p'(\alpha R)}{Hp} \times\sin(\alpha L)\varepsilon_m(I_{m-p}(\alpha d) - I_{m+p}(\alpha d)) \tag{46}$$

$$F_1 = -\frac{4\hat{i}_z\omega\mu_0 R K_p(\alpha R)}{\pi s} \times\sin(\alpha L)\varepsilon_m(I_{m-p}(\alpha d) + I_{m+p}(\alpha d)) \times\frac{I_m(\alpha r_0)}{K_m(\alpha r_0)} + \frac{2\hat{i}_z\omega\mu_0 R^2 K_p'(\alpha R)m}{\alpha^2 r_0^2 a H p} \times\sin(\alpha L)\varepsilon_m(I_{m-p}(\alpha d) - I_{m+p}(\alpha d))\frac{I_m(\alpha r_0)}{K_m(\alpha r_0)I_m'(\alpha r_0)} \times\left(K_m'(\alpha r_0) - \left(\frac{m^2}{a\alpha r_0^2}\left(1-\frac{\alpha^2}{a^2}\right)\frac{I_m(ar_0)}{I_m'(ar_0)} + \frac{\alpha I_m'(ar_0)}{aI_m(ar_0)}\right)K_m(ar_0)\right)^{-1} \tag{47}$$

$$F_2 = \frac{2\hat{i}_z\omega\mu_0 R^2 K_p'(\alpha R)}{Hp}\sin(\alpha L)\varepsilon_m(I_{m-p}(\alpha d) - I_{m+p}(\alpha d))\left(-I_m'(\alpha r_0) + \left(\frac{m^2}{a\alpha r_0^2}\left(1-\frac{\alpha^2}{a^2}\right) \times\frac{I_m(ar_0)}{I_m'(ar_0)} + \frac{\alpha I_m'(ar_0)}{aI_m(ar_0)}\right)I_m(ar_0)\right)\frac{I_m(\alpha r_0)}{K_m(\alpha r_0)I_m'(\alpha r_0)} \times\left(K_m'(\alpha r_0) - \left(\frac{m^2}{a\alpha r_0^2}\left(1-\frac{\alpha^2}{a^2}\right)\frac{I_m(ar_0)}{I_m'(ar_0)} + \frac{\alpha I_m'(ar_0)}{aI_m(ar_0)}\right)K_m(ar_0)\right)^{-1} \tag{48}$$

with $a = (\alpha^2 - \mathrm{j}\omega\mu_0\sigma)^{1/2}$ and:

$$\varepsilon_m = \begin{cases} 1/2 & \text{if } m = 0 \\ 1 & \text{if } m \neq 0. \end{cases} \tag{49}$$

The electromotive force induced in the pick-up coil of the shape presented in figure 6, having one turn and the contour $\Gamma$, is:

$$e = \oint_\Gamma \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} = 4\int_{-L/2}^{L/2}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \times[C_1 I_m(\alpha r_k) + F_1 K_m(\alpha r_k)]\cos(\alpha z)\,\mathrm{d}z \tag{50}$$

where $r_k$ is the pick-up coil radius.

The case when the axis of the conductive cylinder is not the transducer axis is quite common in the practice of non-destructive control, given, on the one hand, the difficulty of perfect centring and, on the other hand, the vibrations of the inspected product during its displacement.

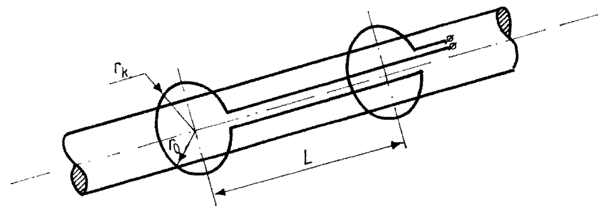


**Figure 6.** Schematic view of pick-up coil.

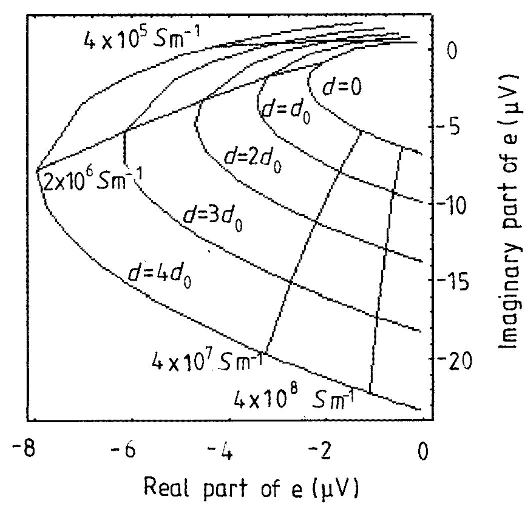


**Figure 7.** The chart of the voltage induced in a single-turn coil for conductivities ranging between $10^5$ and $10^{10}$ S m$^{-1}$, and different distances between the cylinder axis and current source axis ($d_0 = r_k - r_0$).

## 4. Results

The numerical calculations have been conducted for the transducer with rotating magnetic field with the basic diagram presented in figure 6. The magnetic induction in the centre of the torus was $10^{-5}$ T, which implies an amplitude $i_2 = 10$ A for the current in the sheets. From the experimental measurements, the following values of the parameters resulted: $2L = 40$ mm; $2H = 60$ mm; $2R = 40$ mm. The diameter of the studied cylinder was $2r_0 = 4$ mm. The filling factor was: $\eta = (r_0/r_k)^2 = 0.5$.

Figure 7 illustrates the relation (49) in the complex plane for different distances separating the conductive cylinder axis and the transducer axis, the material conductivity changing between $10^5$ and $10^{10}$ S m$^{-1}$ for a frequency of 30 kHz. From the graphics one can notice that an increasing distance between the two symmetry axes results in the increasing electromotive force induced in the transducer for the same value of the material conductivity.

Figure 8 presents the variation of the e.m.f. induced in the transducer pick-up coil calculated from (50) for Inconel 600 with the conductivity $\sigma = 0.1\times10^7$ S m$^{-1}$ and different values of the parameter $d$, the working frequency varying between 1 kHz and 1 MHz.

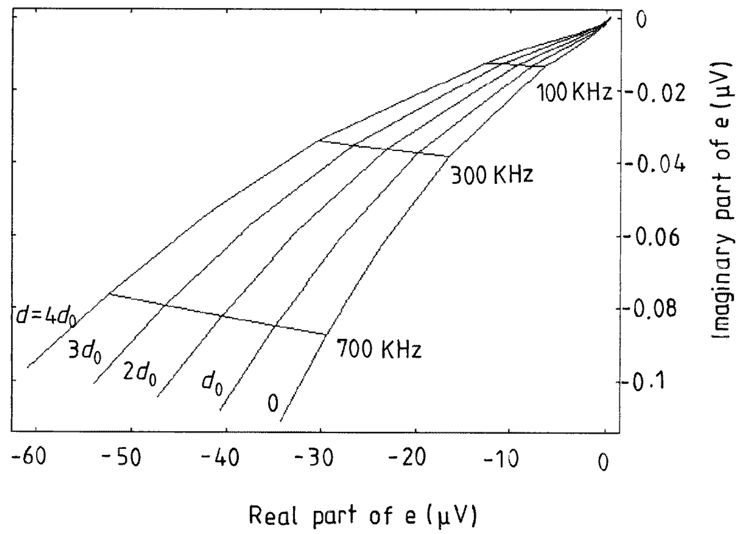


**Figure 8.** The chart of the voltage induced in a single-turn coil for frequencies of the rotating magnetic field ranging between 1 kHz and 1 MHz, and different distances between the cylinder axis and current source axis ($d_0 = r_k - r_0$).

## 5. Discussion and conclusions

This work has presented a method to compute the electromagnetic field created by an arbitrary source of alternating current, placed in the vicinity of an infinitely long conductive cylinder.

Using the dyadic Green function for the free space enables a direct calculation of the electric field in the absence of the conductive cylinder, thus eliminating the necessity to impose boundary conditions on the current source surface. For a certain mode of the solution, the constants are determined depending on two functions given as integrals over the current source volume.

As an application of the obtained general results, the case of a conductive cylinder placed in the rotating magnetic field of a special transducer has been considered.

The application of the general results obtained for the situation when the current source has a more complicated shape is limited by the difficulty of an analytical evaluation of (9), (10), only their numerical evaluation generally being possible.

The generality of the method enables an immediate approach of the situation when the conductive cylinder consists of a certain number of layers of different conductivities.

An approach similar to that of section 2 permits us to calculate the dyadic Green functions of the problem, the electromagnetic field in the material and the air being written in a compact form, as integrals over the current source volume.

The knowledge of these functions enables also the evaluation of the response of the transducer with rotating magnetic field in the case of a conductive cylinder presenting a discontinuity of an arbitrary form.

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